Quark and Gluon Angular Momentum in the Nucleon

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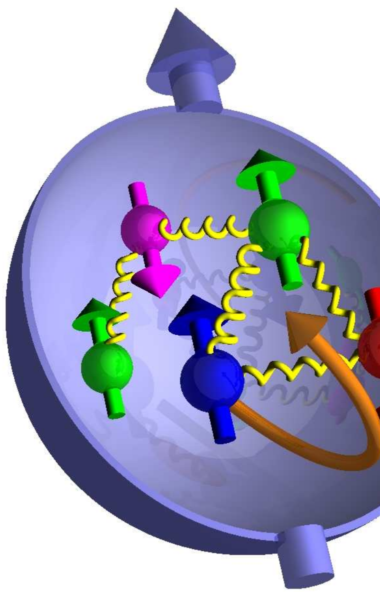
Motivation

- polarized DIS: only $\sim 30\%$ of the proton spin due to quark spins

$\hookrightarrow$ 'spin crisis' $\longrightarrow$ 'spin puzzle', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

$\hookrightarrow$ quest for the remaining 70%

- quark orbital angular momentum (OAM)
- gluon spin
- gluon OAM



$\hookrightarrow$ How are the above quantities defined?

$\hookrightarrow$ How can the above quantities be measured

example: angular momentum in QED

$$\vec{J}_\gamma = \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- use $\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right) = E^j \vec{\nabla} A^j - \left(\vec{E} \cdot \vec{\nabla}\right) \vec{A}$
  and integrate $\int d^3r\, \vec{r} \times \left(\vec{E} \cdot \vec{\nabla}\right) \vec{A}$ by parts

$$\hookrightarrow \vec{J}_\gamma = \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

example (cont.)

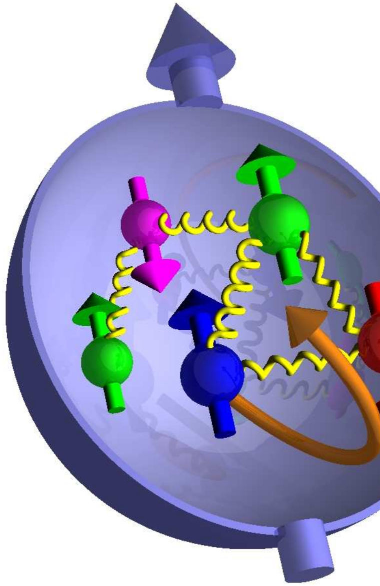
- total angular momentum of isolated system uniquely defined
- ambiguities arise when decomposing $\vec{J}$ into contributions from different constituents
- gauge theories: changing gauge may also shift angular momentum between various degrees of freedom

$\hookrightarrow$ decomposition of angular momentum in general depends on ‘scheme’ (gauge & quantization scheme)

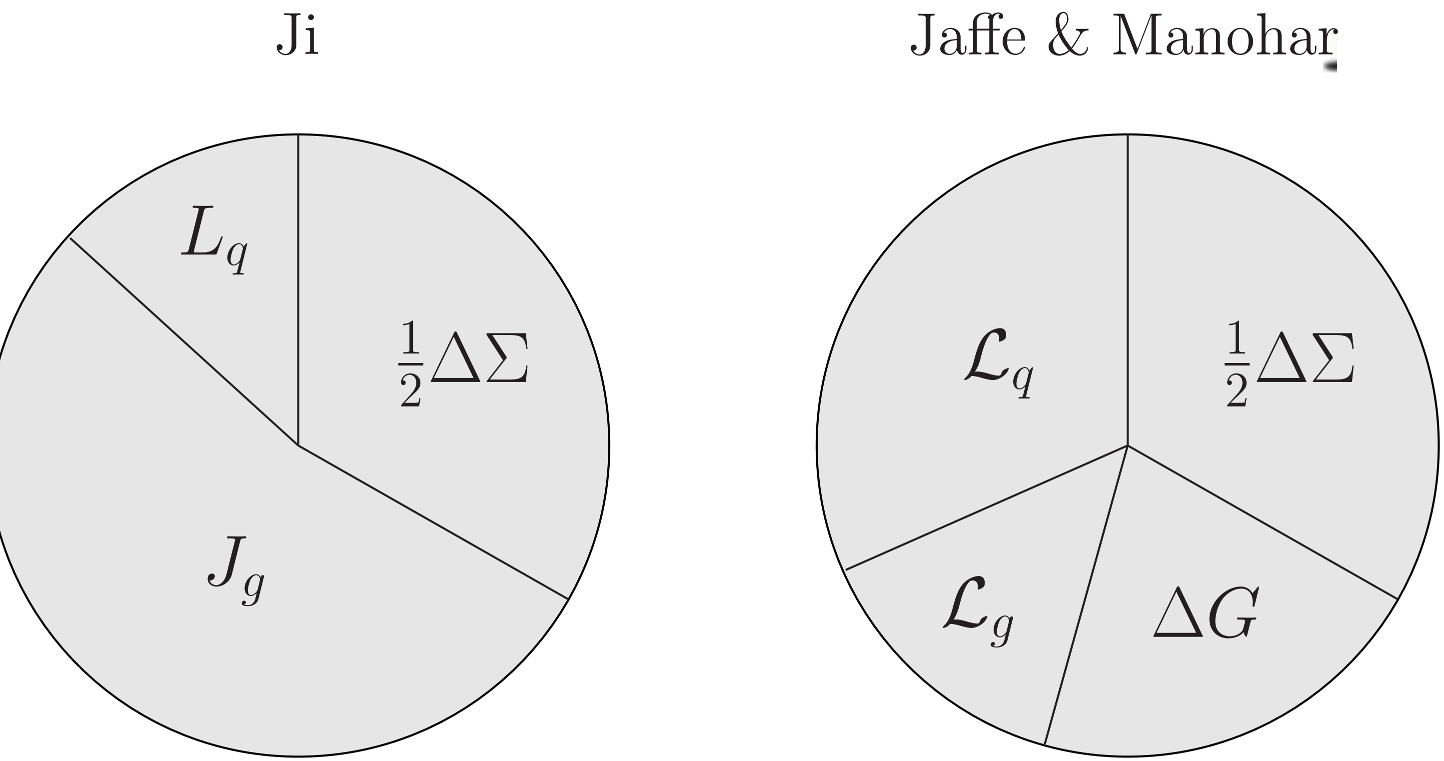
- does not mean that angular momentum decomposition is meaningless, but
- one needs to be aware of this ‘scheme’-dependence in the physical interpretation of exp/lattice/model results in terms of spin vs. OAM
- and, for example, not mix ‘schemes’, e.t.c.

Outline

- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe
- Chen-Goldman decomposition



The nucleon spin pizza(s)



'pizza tre stagioni' 'pizza quattro stagioni'

- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = r^\mu T^{\nu\rho} - r^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

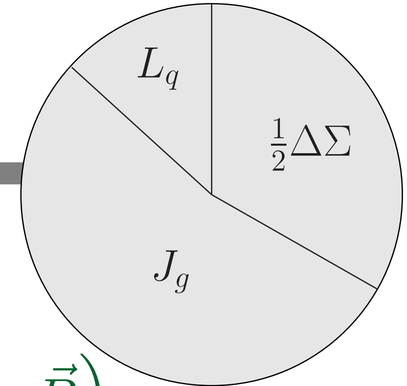
$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3r \partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk}\int d^3r \partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of time derivatives
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on r^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{r}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{r}\times$ does not appear, and can be identified with 'spin'

Ji-decomposition



$\hookrightarrow$ $\vec{J} = \int d^3r \sum_q \left[\frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D}\right) q\right] + \vec{r} \times \left(\vec{E} \times \vec{B}\right)$

- or, with $S^\mu = (0, 0, 0, 1)$

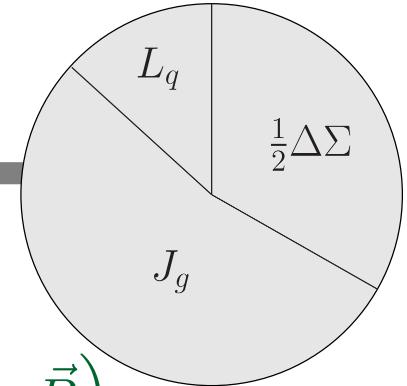
$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

$$\frac{1}{2}\Delta q = \frac{1}{2} \int d^3r \langle P, S| \, q^\dagger(\vec{r}) \Sigma^3 q(\vec{r}) \, |P, S\rangle \qquad \Sigma^3 = i\gamma^1\gamma^2$$

$$L_q = \int d^3r \langle P, S| \, q^\dagger(\vec{r}) \left(\vec{r} \times i\vec{D}\right)^3 q(\vec{r}) \, |P, S\rangle \qquad i\vec{D} = i\vec{\partial} - g\vec{A}$$

$$J_g = \int d^3r \langle P, S| \left[\vec{r} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P, S\rangle$$

Ji-decomposition



$\hookrightarrow$ $\vec{J} = \int d^3r \sum_q \left[\frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D}\right) q\right] + \vec{r} \times \left(\vec{E} \times \vec{B}\right)$
applies to each vector component of nucleon angular momentum, but usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities

- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix element of $q^\dagger \left(\vec{r} \times i\vec{D}\right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- Ji makes no further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) piece

L_q for proton from Ji-relation (lattice)

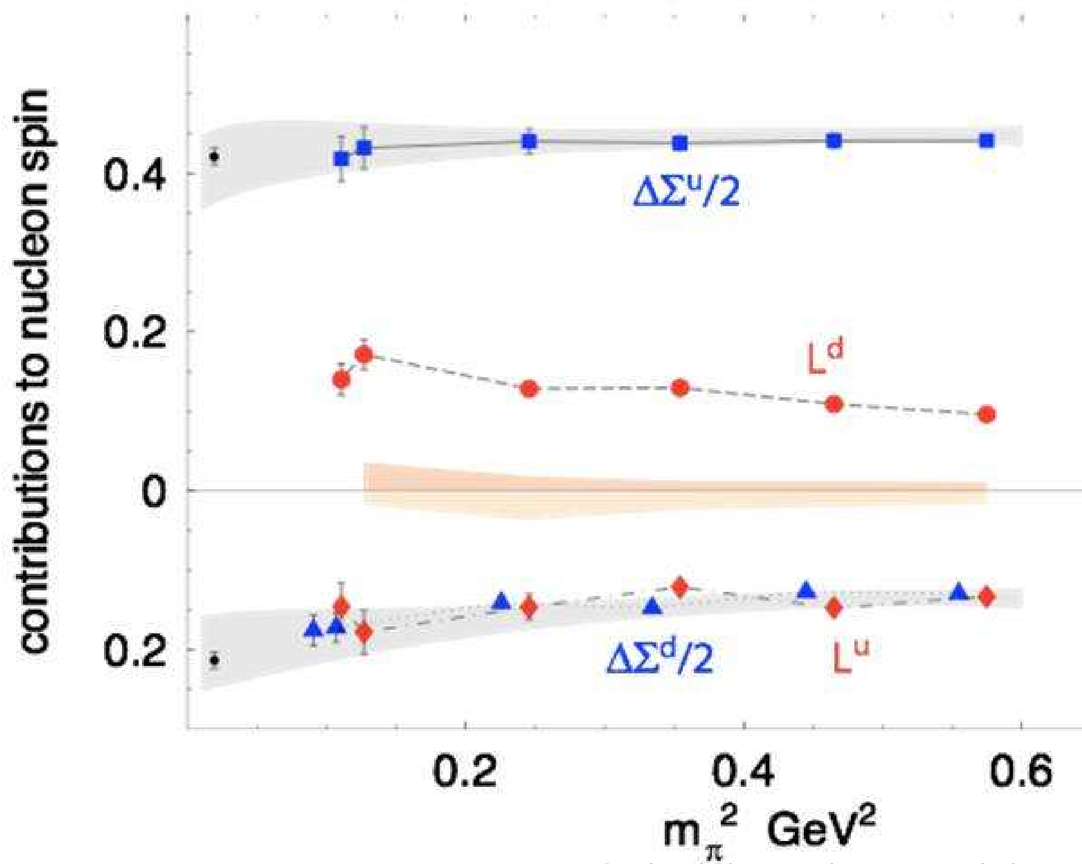
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,\xi,0) + E_q(x,\xi,0) \right] x.$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



The Ji-relation (poor man's derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that L_q can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx\, x \left[H_q(x,\xi,0) + E_q(x,\xi,0) \right]$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)
  - for such a state, $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$ and $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

  $\hookrightarrow$ $\langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$

  $\hookrightarrow$ relate 2^{nd} moment of $\perp$ flavor dipole moment to J_q^x

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  - effect sum of two effects:
    - $\langle T^{++} y\rangle$ for a point-like transversely polarized nucleon
    - $\langle T_q^{++} y\rangle$ for a quark relative to the center of momentum of a transversely polarized nucleon
  - 2^{nd} moment of $\perp$ flavor dipole moment for point-like nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The Ji-relation (poor man's derivation)

- derivation (MB-version):
  - $T_q^{0z} = i\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right)q$
  - since $\psi^\dagger\partial_z\psi$ is even under $y \to -y$, $i\bar{q}\gamma^0\partial^z q$ does not contribute to $\langle T^{0z}y\rangle$
  - $\hookrightarrow$ using $i\partial_0\psi = E\psi$, one finds

$$\begin{aligned}\langle T^{0z}b_y\rangle &= E\int d^3r\psi^\dagger\gamma^0\gamma^z\psi y = E\int d^3r\psi^\dagger\left(\begin{array}{cc}0 & \sigma^z\\ \sigma^z & 0\end{array}\right)\psi y\\ &= \frac{2E}{E+M}\int d^3r\chi^\dagger\sigma^z\sigma^y\chi f(r)(-i)\partial^y f(r)y = \frac{E}{E+M}\int d^3r f^2(r)\end{aligned}$$

  - consider nucleon state with $\vec{p}=0$, i.e. $E=M$ & $\int d^3r f^2(r) = 1$
  - $\hookrightarrow$ 2^{nd} moment of $\perp$ flavor dipole moment $\langle T_q^{++}y\rangle = \langle T^{0z}b_y\rangle = \frac{1}{2}$
  - $\hookrightarrow$ 'overall shift' of nucleon COM yields contribution $\frac{1}{2}\int dx\, xH_q(x,0,0)$ to $\langle T_q^{++}y\rangle$

The Ji-relation (poor man's derivation)

- spherically symmetric wave packet for Dirac particle with $J_x = \frac{1}{2}$ centered around the origin has $\perp$ center of momentum $\frac{1}{M}\langle T_q^{++} b_y \rangle$ not at origin, but at $\frac{1}{2M}$!
- consistent with

$$\frac{1}{2} = \langle J_x \rangle = \langle \left(T^{0z} b^y - T^{0y} b^z \right) \rangle = 2\langle T^{0z} b^y \rangle = \langle T^{++} b^y \rangle$$

  - 'overall shift of $\perp$ COM yields $\langle T_q^{++} b_y \rangle = \frac{1}{2} \int dx \, x H_q(x, 0, 0)$
  - intrinsic distortion adds $\frac{1}{2} \int dx \, x E_q(x, 0, 0)$ to that

Transversely Deformed Distributions and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle \quad = H(x,0,-\boldsymbol{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle \quad = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\boldsymbol{\Delta}_\perp^2).$$

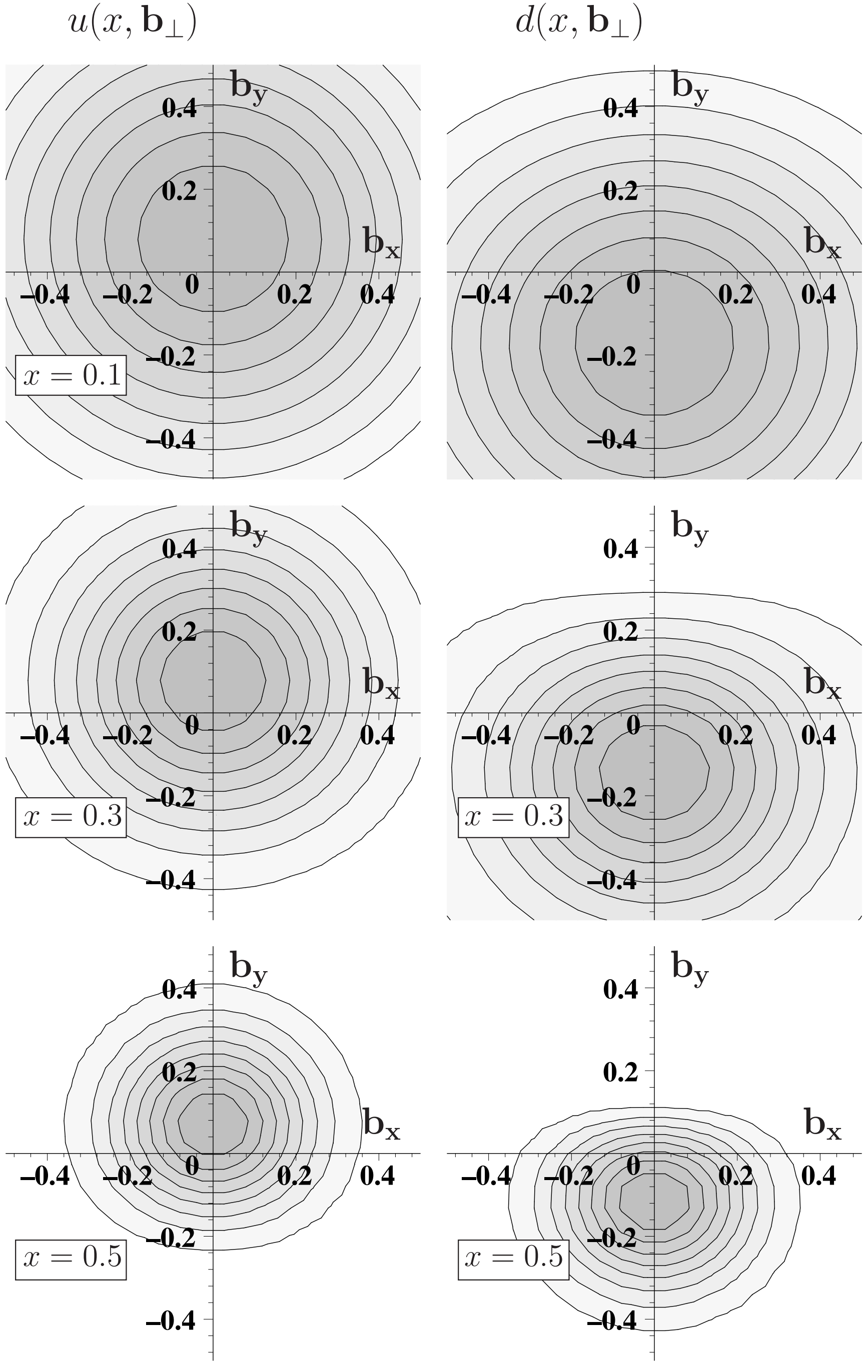
- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

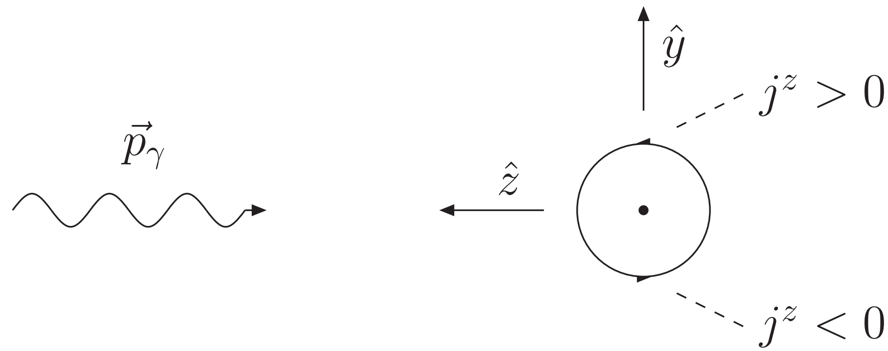
$\hookrightarrow$ unpolarized quark distribution for this state:

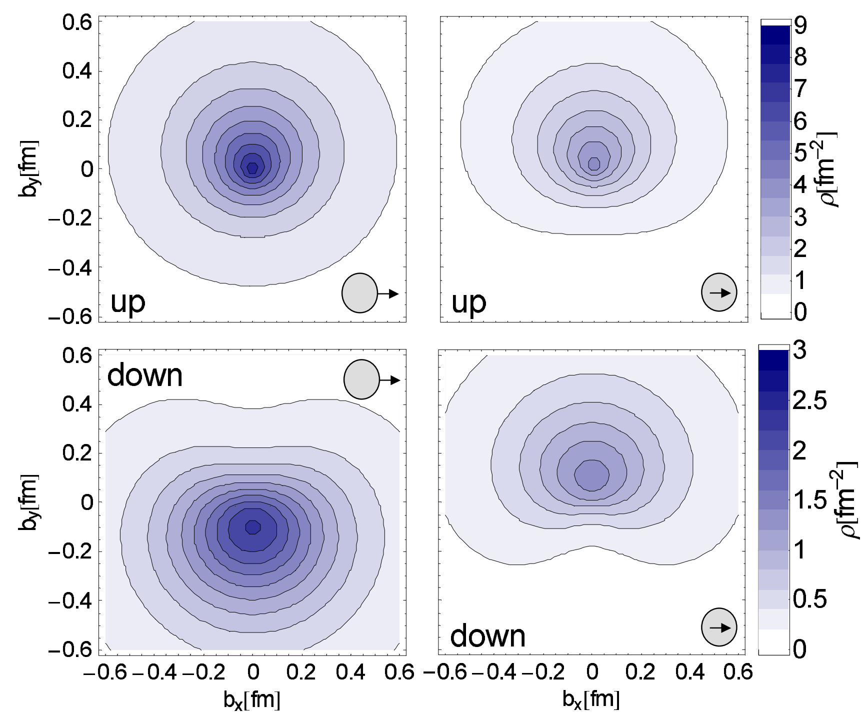
$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} E(x,0,-\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

p polarized in $+\hat{x}$ direction







lattice results (Hägler et al.)

The Ji-relation (poor man's derivation)

- ‘overall shift of $\perp$ COM yields $\langle T_q^{++} b_y \rangle = \frac{1}{2}\int dx\, x H_q(x,0,0)$
- intrinsic distortion adds $\frac{1}{2}\int dx\, x E_q(x,0,0)$ to that

$\hookrightarrow$ Ji relation

$$J_q^x = \frac{1}{2}\int dx\, x \left[H_q(x,0,0) + E_q(x,0,0)\right]$$

- rotational invariance: should apply to each vector component, but parton interpretation (transverse shift) only for $\perp$ pol. nucleon

Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^3\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

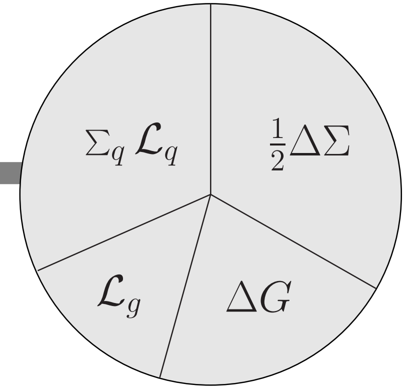
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow$ flux in = flux out)

- use eqs. of motion to get rid of 'time' (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dr^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

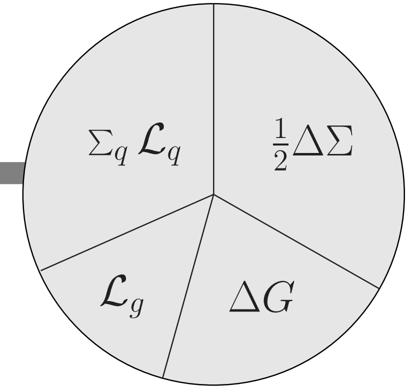
where ($\gamma^+ = \gamma^0 + \gamma^z$)

$$\mathcal{L}_q = \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{r} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle$$

Jaffe/Manohar decomposition



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\vec{p}\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- $J_g - \Delta G \sim \int d^3r F^{+j} \left(\vec{r} \times i\vec{\partial}\right)^z A^j + \psi^\dagger \vec{r} \times g\vec{A}\psi$ Interpretation??

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ vanishes (parity!)

$\hookrightarrow$ L_q identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ (nucleon at rest)

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary (part 1):

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[MB + H. Budhathoki Chhetri (BC), 2009]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

↪ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

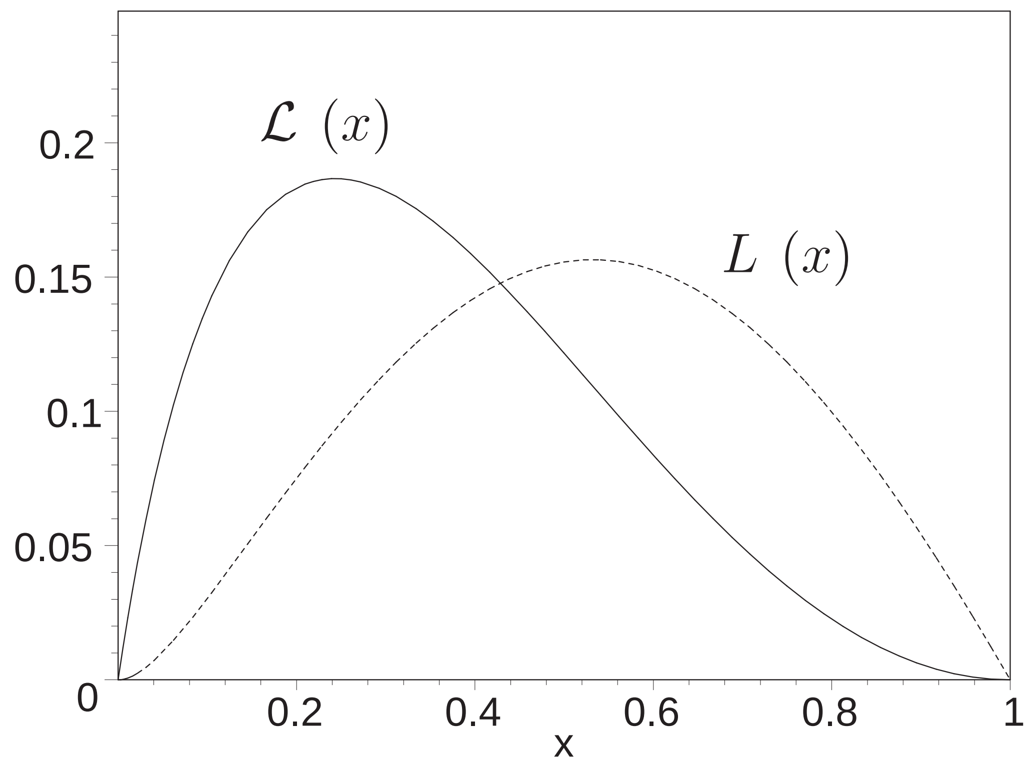
⇝ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is not a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ ‘unintegrated Ji-relation’ does <u>not</u> yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1-ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1+ik}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x}-m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}-1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar
  $\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp (1-x)\left[\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$
- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\,[q(x)+E(x,0,0)] - \frac{1}{2}\Delta q$

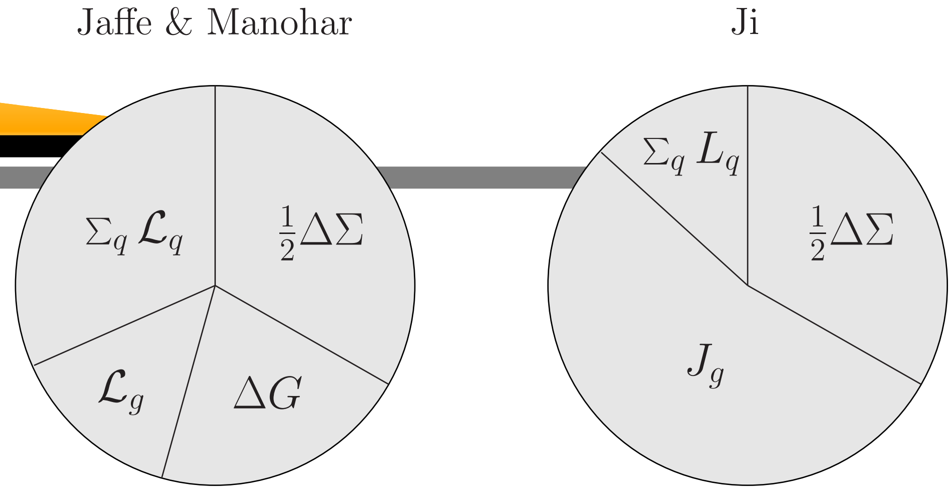
$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and defining $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields
  $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- ↪ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$ (for $j_z = +\frac{1}{2}$)
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- ↪ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d < L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- ↪ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\vec{e}\overleftarrow{p}$ / $\vec{p}\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \frac{1}{2}\sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for $\mathcal{O}(\alpha_s)$ dressed quark

pizza tre e mezzo stagioni

- Chen, Goldman et al.: integrate by parts in J_g only for term involving $\mathbf{A}_{phys}$, where

$$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys} \qquad \text{with} \qquad \nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$$

- $\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q'\right) + S_g' + L_g'$ with Δq as in JM/Ji

$$L_q' = \int d^3x \, \langle P,S| \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}_{pure}\right)^3 q(\vec{x}) \, |P,S\rangle$$

$$S_g' = \int d^3x \, \langle P,S| \left(\vec{E} \times \vec{A}_{phys}\right)^3 |P,S\rangle$$

$$L_g' = \int d^3x \, \langle P,S| \, E^i \left(\vec{x} \times \vec{\nabla}\right)^3 A^i_{phys} \, |P,S\rangle$$

- $i\vec{D}_{pure} = i\vec{\partial} - g\vec{A}_{pure}$
- only $\frac{1}{2}\Delta q$ accessible experimentally

example: angular momentum in QED

- consider now, QED with electrons:

$$\vec{J}_\gamma = \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger \psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

pizza tre e mezzo stagioni

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B.L.T. pizza ?

- Bakker, Leader, Trueman:
- JM only applies for $\mathbf{s} = \hat{\mathbf{p}}$ (helicity sum rule)
- Ji applies to any component, but parton interpretation only for S_z
- For $\mathbf{p} \neq 0$, Ji only applies to helicity
- 'sum rule' $\mathbf{s} \perp \hat{\mathbf{p}}$

$$\frac{1}{2} = \frac{1}{2} \sum_{a \in q, \bar{q}} \int dx h_1^a(x) + \sum_{a \in q, \bar{q}, g} \langle L_{s_T}^a \rangle$$

where $L_{s_T}^a$ component of $\mathbf{L}^a$ along $\mathbf{s}_T$

- note: $\sum_{a \in q, \bar{q}} \int dx h_1^a(x)$ not tensor charge (latter is: '$q - \bar{q}$')
- $\mathbf{L}^a \sim \psi^\dagger \mathbf{k} \times \nabla_k \psi$
- distinction between transversity and transverse spin obscure in two-component formalism used

B.L.T. pizza ?

- ‘B.L.T. sum rule’ $\mathbf{s} \perp \hat{\mathbf{p}}$
  $\frac{1}{2} = \frac{1}{2} \sum_{a \in q,\bar{q}} \int dx h_1^a(x) + \sum_{a \in q,\bar{q},s} \langle L^a_{s_T} \rangle$
- should already be suspicious as $T^{\mu\nu}$ is chirally even ($m_q = 0$) and so should $\vec{J}$...
- $\langle L^a_{s_T} \rangle$ not accessible experimentally, i.e. B.L.T. not experimentally falsifyable, but
- studies (diquark model) under way to test B.L.T. ...